#### MAX PLANCK INSTITUTE FOR INTELLIGENT SYSTEMS

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#### LIE BODIES: A MANIFOLD REPRESENTATION OF 3D HUMAN SHAPE

#### SUPPLEMENTAL MATERIAL

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**Abstract.** This technical report is complementary to [1] and contains proofs, formulas and additional plots. It is identical to the supplemental material submitted to European Conference on Computer Vision (ECCV 2012) on March 2012.

## References

 Freifeld, O., Black, M.J.: Lie Bodies: A Manifold Representation of 3D Human Shape. European Conference on Computer Vision (2012) Lie Bodies: A Manifold Representation of 3D Human Shape

Supplemental Material

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## 1 Proofs (including formulas)

Proof. (Proposition 1)  $I_2 \in G_A$ . To prove closure under composition, let  $A, B \in G_A$ . Note that  $AB[1,0]^T = A[1,0]^T = [1,0]^T$  and det  $AB = \det A \det B > 0$ . Thus,  $AB \in G_A$ . To prove closure under inversion, let  $A \in G_A$ . First, note that  $\det A^{-1} = 1/\det A > 0$ . Second,  $A^{-1} = \begin{pmatrix} 1 & -U/V \\ 0 & 1/V \end{pmatrix}$  and thus  $A^{-1} \in G_A$ .  $\Box$ 

*Proof.* (Proposition 2) Let  $X = [p_1^{(X)}, p_2^{(X)}]$  and  $Y = [p_1^{(Y)}, p_2^{(Y)}]$ . Set  $S = ||p_1^{(Y)}||/||p_1^{(X)}||$ . Without loss of generality we can assume S = 1. Let  $p_2^{(X)} = (x_2^{(X)}, y_2^{(X)})$  and  $p_2^{(Y)} = (x_2^{(Y)}, y_2^{(Y)})$ . Now solve for the unknowns U and V (there exists a unique solution: the y's are positive):

$$\begin{pmatrix} 1 & U \\ 0 & V \end{pmatrix} \begin{pmatrix} x_2^{(X)} \\ y_2^{(X)} \end{pmatrix} = \begin{pmatrix} x_2^{(Y)} \\ y_2^{(Y)} \end{pmatrix} \Rightarrow V = y_2^{(Y)} / y_2^{(X)}, U = (x_2^{(Y)} - x_2^{(X)}) / (y_2^{(X)}) .$$
(1)

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*Proof.* (the fact after Definition 6) Let  $X = [p_1, p_2]$ . First, use any standard technique to find  $R_1 \in SO(3)$  such that  $R_1p_1 = ||p_1|| [1, 0, 0]^T$ . Let  $[x, y, z]^T$  denote the entries of  $R_1p_2$ . Solve for the unknowns c and s in

$$\begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} +\sqrt{y^2 + z^2} \\ 0 \end{pmatrix}$$
(2)

and set  $R_X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{pmatrix} R_1$  (note that  $c^2 + s^2 = 1$ ).

Proof. (Proposition 3) Recall that

$$\exp(A) = \sum_{n=0}^{\infty} \frac{1}{n!} A^n .$$
(3)

First, assume v = 0. So  $A^2 = AA$  is the zero matrix and thus so is  $A^k$  for every  $k \ge 2$ . By Eq. (3),

$$\exp(A) = \exp\left(\begin{pmatrix} 0 & u \\ 0 & 0 \end{pmatrix}\right) = I + \begin{pmatrix} 0 & u \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix} \triangleq \begin{pmatrix} 1 & U \\ 0 & V \end{pmatrix} \in G_A .$$
(4)

If  $v \neq 0$ , induction shows that

$$A^{n} = \begin{pmatrix} 0 & u \\ 0 & v \end{pmatrix}^{n} = \begin{pmatrix} 0 & uv^{n-1} \\ 0 & v^{n} \end{pmatrix} .$$
 (5)

Substituting this into Eq. (3),

$$\exp(A) = \exp\left(\begin{pmatrix} 0 & u \\ 0 & v \end{pmatrix}\right) = \begin{pmatrix} 1 & \frac{u}{v} \left(e^{v} - 1\right) \\ 0 & e^{v} \end{pmatrix} \triangleq \begin{pmatrix} 1 & U \\ 0 & V \end{pmatrix} \in G_A .$$
(6)

The two cases taken together imply a bijection  $(u, v) \mapsto (U, V)$ ,  $\mathbb{R}^2 \to \mathbb{R} \times \mathbb{R}^+$ , and thus exp :  $\mathfrak{g}_A \to : G_A$  is bijective too. Finally, For computing the log, set  $v = \log(V)$ . If V = 1, set u = U. Otherwise, set u = Uv/(V - 1).

# 2 Additional Plots for Prediction of Measurements

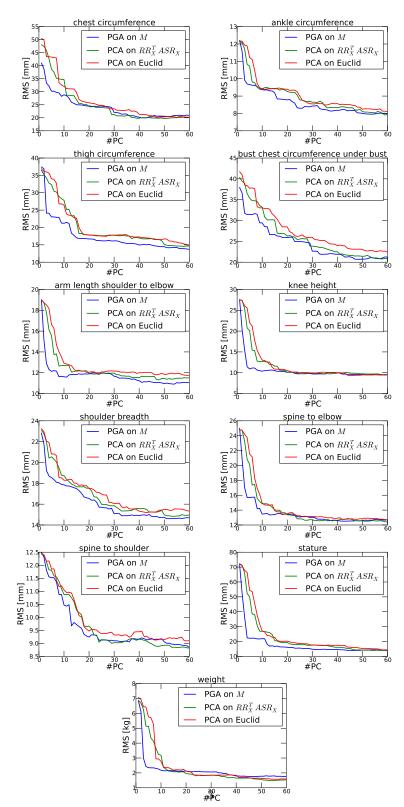


Figure 1: **Measurement prediction.** Linear prediction of body measurements from shape coefficients; RMS error as a function of the number of coefficients