## LIE BODIES:

# A MANIFOLD REPRESENTATION OF 3D HUMAN SHAPE 

## SUPPLEMENTAL MATERIAL

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#### Abstract

This technical report is complementary to [1] and contains proofs, formulas and additional plots. It is identical to the supplemental material submitted to European Conference on Computer Vision (ECCV 2012) on March 2012.


## References

[1] Freifeld, O., Black, M.J.: Lie Bodies: A Manifold Representation of 3D Human Shape. European Conference on Computer Vision (2012)

# Lie Bodies: <br> A Manifold Representation of 3D Human Shape 

Supplemental Material

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1 Proofs (including formulas)
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## 1 Proofs (including formulas)

Proof. (Proposition 1) $I_{2} \in G_{A}$. To prove closure under composition, let $A, B \in$ $G_{A}$. Note that $A B[1,0]^{T}=A[1,0]^{T}=[1,0]^{T}$ and $\operatorname{det} A B=\operatorname{det} A \operatorname{det} B>0$. Thus, $A B \in G_{A}$. To prove closure under inversion, let $A \in G_{A}$. First, note that $\operatorname{det} A^{-1}=1 / \operatorname{det} A>0$. Second, $A^{-1}=\left(\begin{array}{cc}1 & -U / V \\ 0 & 1 / V\end{array}\right)$ and thus $A^{-1} \in G_{A}$.
Proof. (Proposition 2) Let $X=\left[p_{1}^{(X)}, p_{2}^{(X)}\right]$ and $Y=\left[p_{1}^{(Y)}, p_{2}^{(Y)}\right]$. Set $S=$ $\left\|p_{1}^{(Y)}\right\| /\left\|p_{1}^{(X)}\right\|$. Without loss of generality we can assume $S=1$. Let $p_{2}^{(X)}=$ $\left(x_{2}^{(X)}, y_{2}^{(X)}\right)$ and $p_{2}^{(Y)}=\left(x_{2}^{(Y)}, y_{2}^{(Y)}\right)$. Now solve for the unknowns $U$ and $V$ (there exists a unique solution: the $y$ 's are positive):

$$
\left(\begin{array}{ll}
1 & U  \tag{1}\\
0 & V
\end{array}\right)\binom{x_{2}^{(X)}}{y_{2}^{(X)}}=\binom{x_{2}^{(Y)}}{y_{2}^{(Y)}} \Rightarrow V=y_{2}^{(Y)} / y_{2}^{(X)}, U=\left(x_{2}^{(Y)}-x_{2}^{(X)}\right) /\left(y_{2}^{(X)}\right) .
$$

Proof. (the fact after Definition 6) Let $X=\left[p_{1}, p_{2}\right]$. First, use any standard technique to find $R_{1} \in \mathrm{SO}(3)$ such that $R_{1} p_{1}=\left\|p_{1}\right\|[1,0,0]^{T}$. Let $[x, y, z]^{T}$ denote the entries of $R_{1} p_{2}$. Solve for the unknowns $c$ and $s$ in

$$
\left(\begin{array}{cc}
c & -s  \tag{2}\\
s & c
\end{array}\right)\binom{y}{z}=\binom{+\sqrt{y^{2}+z^{2}}}{0}
$$

and set $R_{X}=\left(\begin{array}{ccc}1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c\end{array}\right) R_{1}\left(\right.$ note that $\left.c^{2}+s^{2}=1\right)$.

Proof. (Proposition 3) Recall that

$$
\begin{equation*}
\exp (A)=\sum_{n=0}^{\infty} \frac{1}{n!} A^{n} \tag{3}
\end{equation*}
$$

First, assume $v=0$. So $A^{2}=A A$ is the zero matrix and thus so is $A^{k}$ for every $k \geq 2$. By Eq. (3),

$$
\exp (A)=\exp \left(\left(\begin{array}{ll}
0 & u  \tag{4}\\
0 & 0
\end{array}\right)\right)=I+\left(\begin{array}{ll}
0 & u \\
0 & 0
\end{array}\right)=\left(\begin{array}{ll}
1 & u \\
0 & 1
\end{array}\right) \triangleq\left(\begin{array}{ll}
1 & U \\
0 & V
\end{array}\right) \in G_{A} .
$$

If $v \neq 0$, induction shows that

$$
A^{n}=\left(\begin{array}{ll}
0 & u  \tag{5}\\
0 & v
\end{array}\right)^{n}=\left(\begin{array}{cc}
0 & u v^{n-1} \\
0 & v^{n}
\end{array}\right) .
$$

Substituting this into Eq. (3),

$$
\exp (A)=\exp \left(\left(\begin{array}{ll}
0 & u  \tag{6}\\
0 & v
\end{array}\right)\right)=\left(\begin{array}{cc}
1 & \frac{u}{v}\left(e^{v}-1\right) \\
0 & e^{v}
\end{array}\right) \triangleq\left(\begin{array}{ll}
1 & U \\
0 & V
\end{array}\right) \in G_{A} .
$$

The two cases taken together imply a bijection $(u, v) \mapsto(U, V), \mathbb{R}^{2} \rightarrow \mathbb{R} \times \mathbb{R}^{+}$, and thus $\exp : \mathfrak{g}_{A} \rightarrow: G_{A}$ is bijective too. Finally, For computing the log, set $v=\log (V)$. If $V=1$, set $u=U$. Otherwise, set $u=U v /(V-1)$.

## 2 Additional Plots for Prediction of Measurements



Figure 1: Measurement prediction. Linear prediction of body measurements from shape coefficients; RMS error as a function of the number of coefficients

