MAX PLANCK INSTITUTE FOR INTELLIGENT SYSTEMS

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LIE BODIES: A MANIFOLD REPRESENTATION OF 3D HUMAN SHAPE

SUPPLEMENTAL MATERIAL

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Abstract. This technical report is complementary to [1] and contains proofs, formulas and additional plots. It is identical to the supplemental material submitted to European Conference on Computer Vision (ECCV 2012) on March 2012.

References

[1] Freifeld, O., Black, M.J.: Lie Bodies: A Manifold Representation of 3D Human Shape. European Conference on Computer Vision (2012)

Lie Bodies: A Manifold Representation of 3D Human Shape

Supplemental Material

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Contents

1 Proofs (including formulas)

Proof. (Proposition 1) $I_2 \in G_A$. To prove closure under composition, let $A, B \in G_A$. Note that $AB[1,0]^T = A[1,0]^T = [1,0]^T$ and $\det AB = \det A \det B > 0$. Thus, $AB \in G_A$. To prove closure under inversion, let $A \in G_A$. First, note that $\det A^{-1} = 1/\det A > 0$. Second, $A^{-1} = \begin{pmatrix} 1 & -U/V \\ 0 & 1/V \end{pmatrix}$ and thus $A^{-1} \in G_A$. □

Proof. (Proposition 2) Let $X = [p_1^{(X)}, p_2^{(X)}]$ and $Y = [p_1^{(Y)}, p_2^{(Y)}]$. Set $S = ||p_1^{(Y)}||/||p_1^{(X)}||$. Without loss of generality we can assume $S = 1$. Let $p_2^{(X)} = (x_2^{(X)}, y_2^{(X)})$ and $p_2^{(Y)} = (x_2^{(Y)}, y_2^{(Y)})$. Now solve for the unknowns U and V (there exists a unique solution: the y's are positive):

$$\begin{pmatrix} 1 & U \\ 0 & V \end{pmatrix} \begin{pmatrix} x_2^{(X)} \\ y_2^{(X)} \end{pmatrix} = \begin{pmatrix} x_2^{(Y)} \\ y_2^{(Y)} \end{pmatrix} \Rightarrow V = y_2^{(Y)}/y_2^{(X)}, U = (x_2^{(Y)} - x_2^{(X)})/(y_2^{(X)}) \ . \quad (1)$$

□

Proof. (the fact after Definition 6) Let $X = [p_1, p_2]$. First, use any standard technique to find $R_1 \in \mathrm{SO}(3)$ such that $R_1 p_1 = \|p_1\| [1, 0, 0]^T$. Let $[x, y, z]^T$ denote the entries of $R_1 p_2$. Solve for the unknowns c and s in

$$\begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} +\sqrt{y^2 + z^2} \\ 0 \end{pmatrix} \tag{2}$$

and set $R_X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{pmatrix} R_1$ (note that $c^2 + s^2 = 1$).

□

Proof. (Proposition 3) Recall that

$$\exp(A) = \sum_{n=0}^{\infty} \frac{1}{n!} A^n \ . \tag{3}$$

First, assume $v = 0$. So $A^2 = AA$ is the zero matrix and thus so is A^k for every $k \geq 2$. By Eq. (3),

$$\exp(A) = \exp\left(\begin{pmatrix} 0 & u \\ 0 & 0 \end{pmatrix}\right) = I + \begin{pmatrix} 0 & u \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix} \triangleq \begin{pmatrix} 1 & U \\ 0 & V \end{pmatrix} \in G_A \ . \tag{4}$$

If $v \neq 0$, induction shows that

$$A^n = \begin{pmatrix} 0 & u \\ 0 & v \end{pmatrix}^n = \begin{pmatrix} 0 & uv^{n-1} \\ 0 & v^n \end{pmatrix} \ . \tag{5}$$

Substituting this into Eq. (3),

$$\exp(A) = \exp\left(\begin{pmatrix} 0 & u \\ 0 & v \end{pmatrix}\right) = \begin{pmatrix} 1 & \frac{u}{v}(e^v - 1) \\ 0 & e^v \end{pmatrix} \triangleq \begin{pmatrix} 1 & U \\ 0 & V \end{pmatrix} \in G_A \ . \tag{6}$$

The two cases taken together imply a bijection $(u, v) \mapsto (U, V)$, $\mathbb{R}^2 \to \mathbb{R} \times \mathbb{R}^+$, and thus $\exp : \mathfrak{g}_A \to: G_A$ is bijective too. Finally, For computing the log, set $v = \log(V)$. If $V = 1$, set $u = U$. Otherwise, set $u = Uv/(V - 1)$.

□

2 Additional Plots for Prediction of Measurements

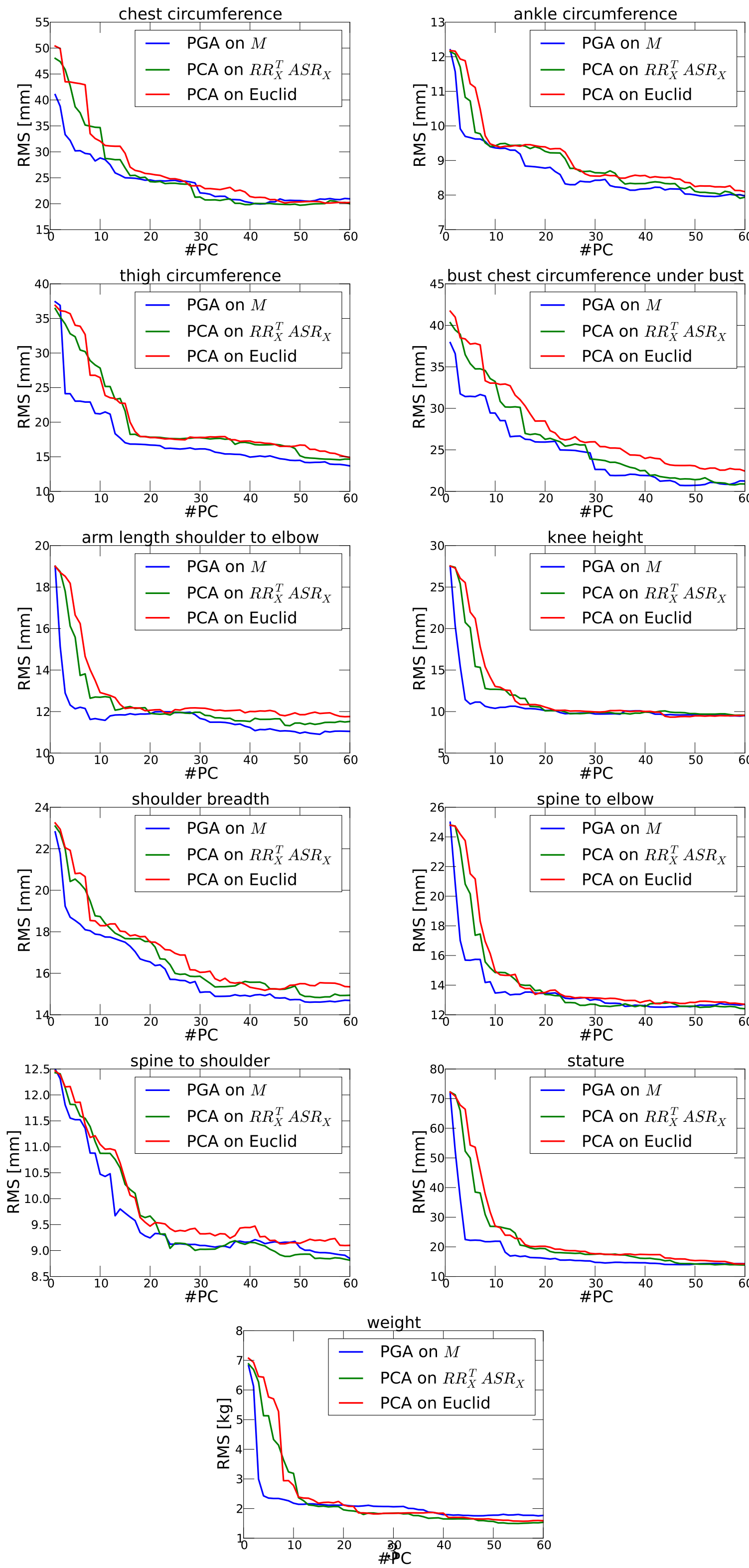

Figure 1: **Measurement prediction.** Linear prediction of body measurements from shape coefficients; RMS error as a function of the number of coefficients