Lie Bodies: A Manifold Representation of 3D Human Shape

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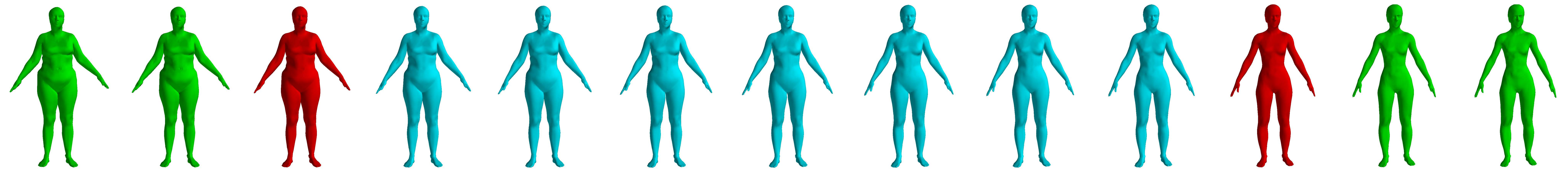
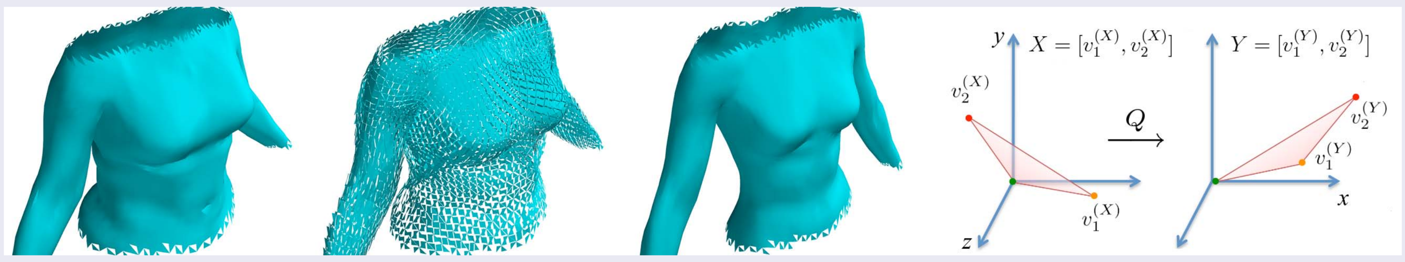
Scans of real people (CAESAR[1] dataset)
Geodesic interpolation
Geodesic extrapolation



Introduction

Statistical deformable shape models have wide application in computer vision, graphics and biometrics

Deformation of triangles are the basic units



Goal:

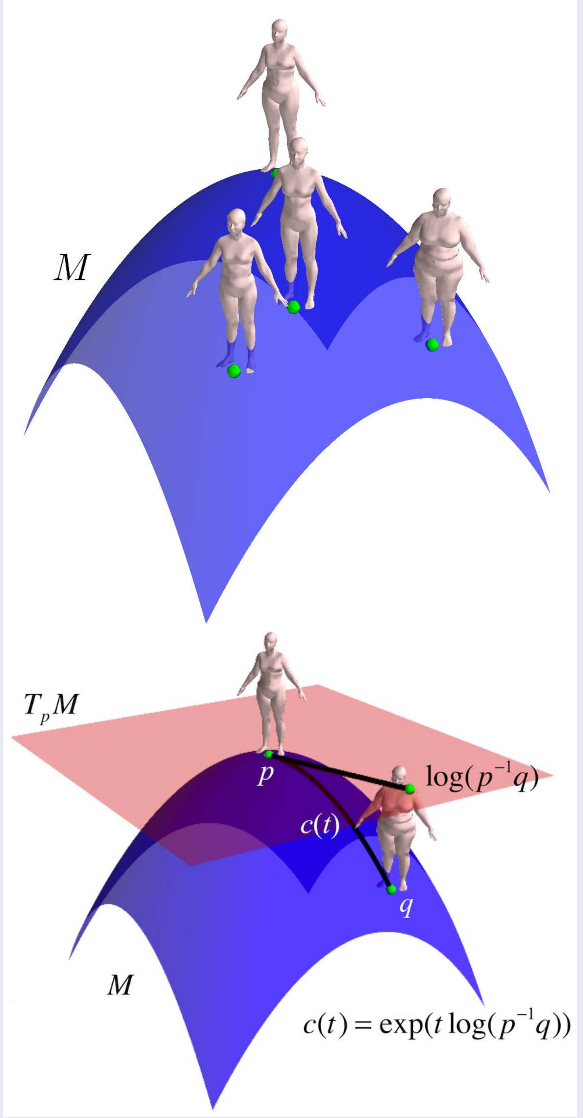
Effective statistical modeling of shape variability

Problems:

- $Y = QX$. $X, Y \in \mathbb{R}^{3\times 2}$, $Q \in \mathbb{R}^{3\times 3}$. 6 constraints, 9 DoF. $Q = ?$
- Issues with existing models: redundant DoF; assume Euclidean geometry while Q matrices do not form a linear space; synthesis is prone to inconsistency (e.g., $\det Q < 0$ or $\det Q = 0$)

Solution

A shape is a point on a non-linear manifold, $M \triangleq G_T^{N_T}$. G_T is novel 6D Lie group



Advantages

- Consistency
- No redundant DoF $\Rightarrow$ less noise (In CAESAR, the *Euclidean* variance in our method was 1.68 smaller)
- A principled definition of distance
- Closed-form formulas: $\exp$, $\log$ and geodesics

Lie Groups and Lie Algebras

- Key concept: The tangent space
- Transitions: $\exp$ & $\log$
- A geodesic distance and a geodesic path:

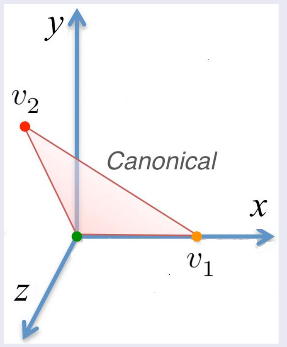
$$d(p,q) = \left\|\log(p^{-1}q)\right\|_F$$
$$c(t) = p\exp(t\log(p^{-1}q))$$

[1]Robinette, et al. : Civilian American and European Surface Anthropometry Resource (CAESAR) final report. AFRL-HE-WP-TR-2002-0169, US AFRL (2002)

Canonical Triangles

v_1 has the same the direction as the positive x-axis
v_2 lies in the the upper half of the xy-plane

If a triangle X is not canonical, we can always find a rotation matrix, R_X, such that $R_X X$ is canonical

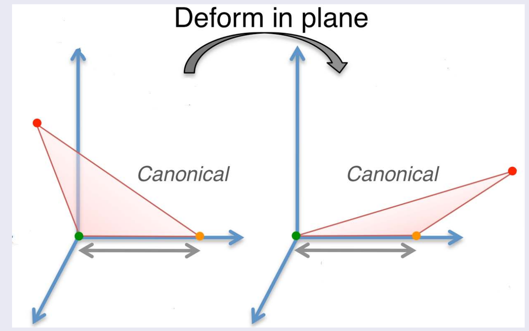


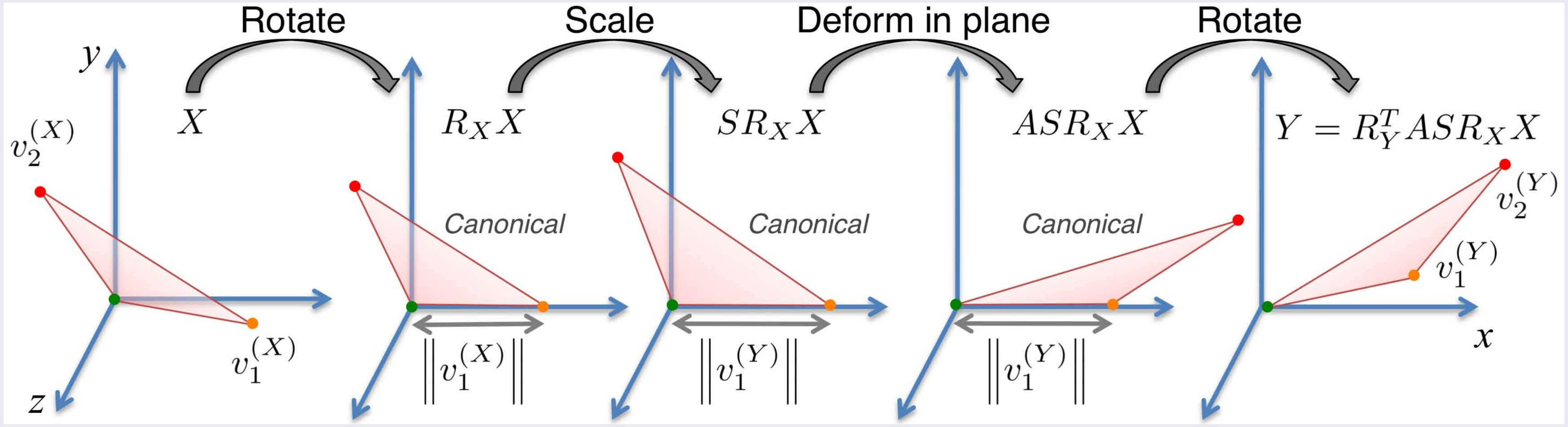
G_T: A Lie Group Of Triangle Deformations

$$G_T \triangleq \underset{\text{Rotations}}{\mathrm{SO}(3)} \times \underset{\text{NEW}}{G_A} \times \underset{\text{Scaling}}{G_S}$$

Composition: $((R_1, A_1, S_1), (R_2, A_2, S_2)) \mapsto (R_1R_2, A_1A_2, S_1S_3)$

G_A preserves canonization of triangles as well as well as the length of v_1





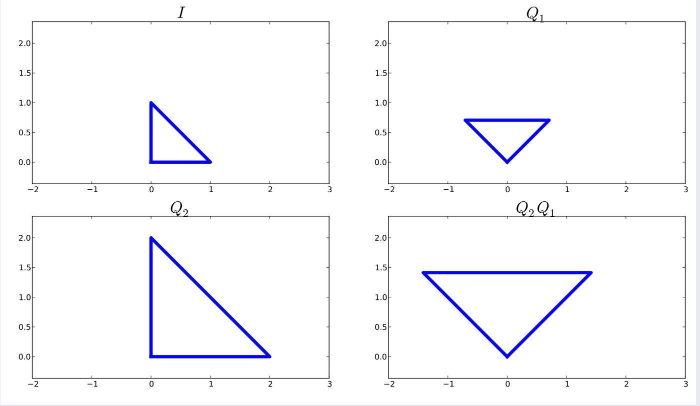
$$R_Y Y = ASR_X \Rightarrow Y = R_Y^T ASR_X X \qquad R \triangleq R_Y^T R_X$$

$$Y = RR_X^T ASR_X X$$

Left-Invariant Metric

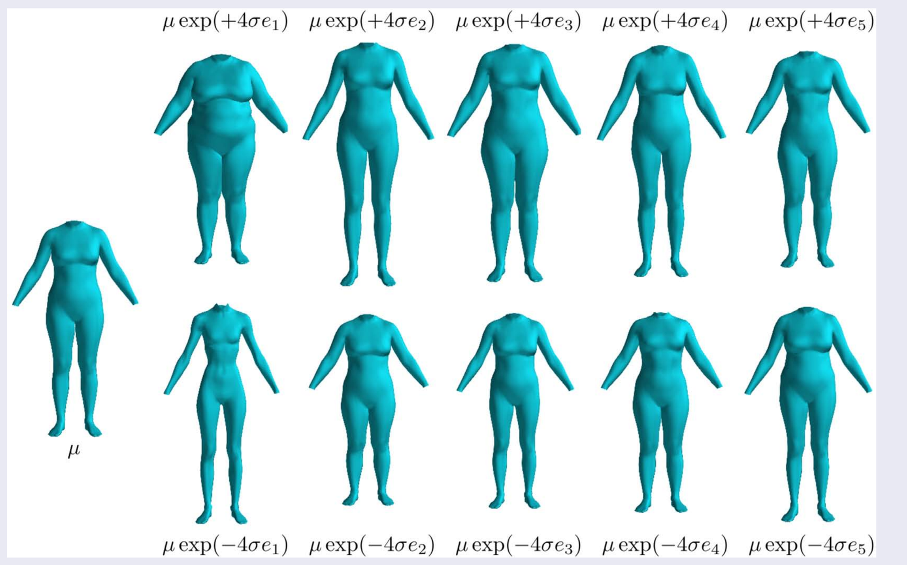
$d(p_1,p_2) = d(p_3p_1, p_3p_2)\ \forall p_1,p_2,p_3 \in M$
e.g.

$$d(I, Q_1) = d(Q_2I, Q_2Q_1)$$
$$d(I, \text{rotation}) = d(\text{scale}, \text{scale } \& \text{ rotation})$$



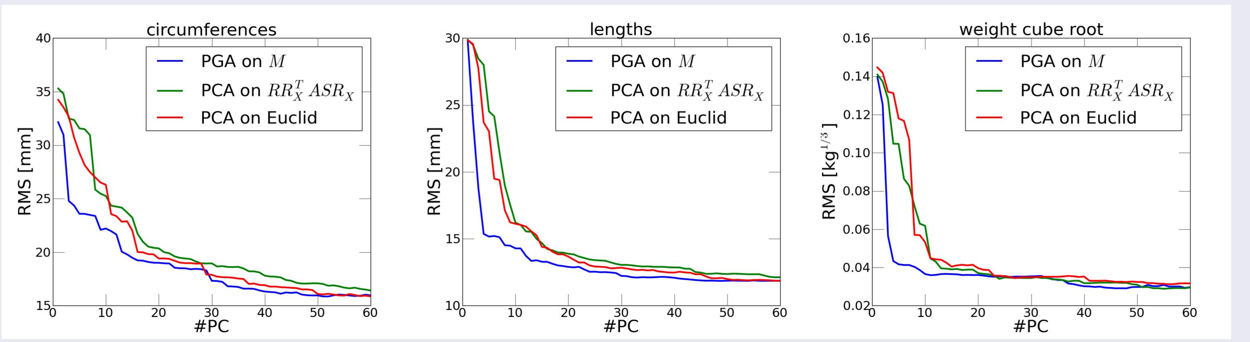
Statistics on Manifolds and Principal Geodesic Analysis

- Data: $\{p_1, p_2, \dots, p_n\} \subset M$
- Intrinsic Mean: $\arg\min_{\mu\in M} \sum_{i=1}^n d(p_i,\mu)^2$
- $g_i \triangleq \mu^{-1}p_i$ $\Rightarrow \log g_i \in T_IM \cong T_\mu M$
- Eigenvectors $\{e_k\}_{k=1}^K \subset T_\mu M$
- Synthesis: $\mu\exp(\sum_{k=1}^K \alpha_k e_k)$



Predicting Biometric Measurements

Linear regression from subspace coefficients to measurements



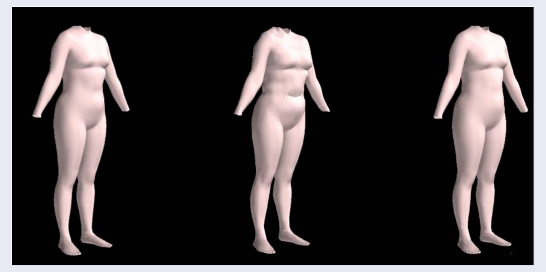
Mesh Reconstruction: Edges

Table: Mean edge RMS for mesh reconstruction using a subspace

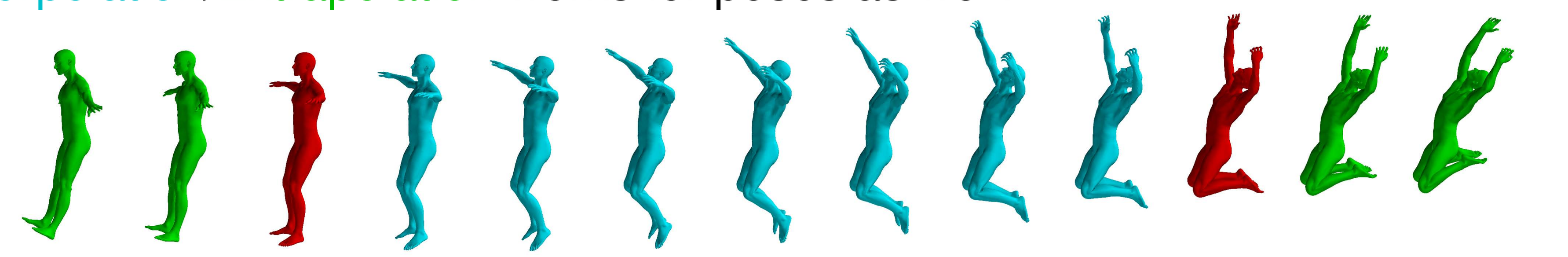
#PC's	5	10	15	20	25	30	35	40	45	50	100
Euclidean method. Ave. RMS [mm]	2.71	2.53	2.43	2.34	2.28	2.23	2.19	2.15	2.11	2.08	1.91
Our method. Ave. RMS [mm]	2.57	2.43	2.32	2.26	2.21	2.17	2.12	2.09	2.06	2.03	1.88

Mesh Reconstruction: Human Shape Perception

- 20 PCA/PGA coefficients
- 300 examples, each seen by 10 people
- Our approach was preferred 56% of the time
  $\Pr(M \text{ won}|\text{majority was achieved}) = 0.69$
  (note that the PGA subspace is a maximizer of the captured variance and need not be the minimizer of the reconstruction MSE)



Interpolation/Extrapolation works for poses as well:



For more extreme poses, use skeleton info. Red shapes: http://tosca.cs.technion.ac.il