Supplement – From Deformations to Parts: Motion-based Segmentation of 3D Objects

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1 Likelihood derivation

Here, we will denote Y_{jk} and $X_{b_j k}$ simply as Y and X.

$$Y|X \sim \mathcal{MN}(AX, \Sigma, \mathbf{I}) \tag{1}$$

It can be shown that [1]

$$p(Y|X,\Sigma) = \int p(Y, A|X, \Sigma)dA = \frac{|K|^{3/2}}{(2\pi)^{3N/2}|\Sigma|^{N/2}|S_{xx}|^{3/2}} exp\{-\frac{1}{2}tr(\Sigma^{-1}S_{y|x})\} \tag{2}$$

and

$$S_{xx} = XX^T + K \tag{3}$$

$$S_{yx} = YX^T + MK \tag{4}$$

$$S_{y|x} = YY^T + MKM^T - S_{yx}(S_{xx})^{-1}S_{yx}^T \tag{5}$$

Finally, the marginal likelihood is given by

$$p(Y|X) = \int p(Y|X,\Sigma)p(\Sigma|n_0, S_0)d\Sigma \tag{6}$$

$$= \int \frac{|K|^{3/2}}{(2\pi)^{3N/2}|\Sigma|^{N/2}|S_{xx}|^{3/2}} exp\{-\frac{1}{2}tr(\Sigma^{-1}S_{y|x})\} \tag{7}$$

$$\frac{|S_0|^{n_0/2}|\Sigma|^{-(4+n_0)/2}}{2^{3n_0/2}\Gamma_3(n_0/2)} exp\{-\frac{1}{2}tr(\Sigma^{-1}S_0)\}d\Sigma \tag{8}$$

$$p(Y|X) = \int \frac{|K|^{3/2}|S_0|^{n_0/2}|\Sigma|^{-(4+n_0)/2}}{(2\pi)^{3N/2}|\Sigma|^{N/2}|S_{xx}|^{3/2}2^{3n_0/2}\Gamma_3(n_0/2)} exp\{-\frac{1}{2}tr(\Sigma^{-1}(S_{y|x}+S_0))\}d\Sigma \tag{9}$$

$$p(Y|X) = \frac{|K|^{3/2}|S_0|^{n_0/2}}{(2\pi)^{3N/2}|S_{xx}|^{3/2}2^{3n_0/2}\Gamma_3(n_0/2)} \int |\Sigma|^{-(3+N+n_0+1)/2} exp\{-\frac{1}{2}tr(\Sigma^{-1}(S_{y|x}+S_0))\} \tag{10}$$

$$p(Y|X) = \frac{|K|^{3/2}|S_0|^{n_0/2}2^{(N+n_0)3/2}\Gamma_3((N+n_0)/2)}{|2\pi|^{3N/2}|S_{xx}|^{3/2}2^{3n_0/2}\Gamma_3(n_0/2)|S_0+S_{y|x}|^{(N+n_0)/2}} \int IW(N+n_0, S_{y|x}+S_0)d\Sigma \tag{11}$$

The part likelihood is then given by

$$p(Y|X,K,n_0,S_0) = \frac{|K|^{\frac{3}{2}}|S_0|^{\frac{n_0}{2}}\Gamma_3\left(\frac{N+n_0}{2}\right)}{\pi^{\frac{3N}{2}}|S_{xx}|^{\frac{3}{2}}|S_0+S_{y|x}|^{\frac{(N+n_0)}{2}}\Gamma_3(\frac{n_0}{2})} \tag{12}$$

References

[1] E. B. Fox. *Bayesian Nonparametric Learning of Complex Dynamical Phenomena*. PhD thesis, Massachusetts Institute of Technology, Cambridge, MA, 2009.